MATH 106 CALCULUS I FOR BIO. & SOC. SCI. FALL 2012

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Homework 5.

Please show all your work.

(1) Consider the function $f(x) = |x|$.

(a) Compute

$$\lim_{h\to 0^-} \frac{f(h)-f(0)}{h} \text{ and } \lim_{h\to 0^+} \frac{f(h)-f(0)}{h}.$$

(b) Use the above to conclude that $f'(0)$ does not exist even though f is continuous everywhere.

(2) Compute the derivatives of the following functions.

(a)

$$a(x) = 2x^2\cos(\frac{\pi}{2}) + ex^3.$$

(b)

$$b(t) = \sqrt{t} + \frac{1}{\sqrt{t}}.$$

(c)

$$c(x) = \frac{x+1}{x^2-2}.$$

(d)

$$d(x) = \sqrt{x}(x^2+1).$$

(e)

$$e(s) = \frac{s^{1/2} - s^{-1/2}}{s^2+1}.$$

(3) Find the point or points in the parabola with equation $y = 3x^2 - 6x - 2$ where the tangent line is horizontal.

(4) Consider the function $g(x)$ defined by

$$g(x) = \frac{2x^3}{3} + \frac{5x^2}{2} + x - 2.$$

Find the point or points (x_0, y_0) in the graph of $g(x)$ in such a way that the tangent line to the graph of $g(x)$ at the point (x_0, y_0) is parallel to the line $y = 4x - 2$.

(5) Let $h(x) = x^2 + 1$. Find the point or points (x_0, y_0) in the graph of $h(x)$ in such a way that the tangent line to the graph of $h(x)$ at the point (x_0, y_0) passes through the point $(0, 0)$.

(6) Use the limit definition of the derivative to show that

$$\frac{d}{dx}\cos(x) = -\sin(x).$$

Remark: No credit will be given to computations that don't use the limit definition of the derivative.

(7) A ball is thrown upwards from the top of a building that is H feet tall. The ball is given an initial velocity of 16 ft/sec. The distance from the ground $s(t)$ (measured in in feet), t seconds after the ball was thrown, is given by the equation

$$s(t) = H + 16t - 16t^2.$$

Suppose it takes 5 seconds for the ball to hit the ground, answer the following questions.

(a) Find the value of H; that is, find how tall the building is.

(b) How long does it take for the ball to reach its maximum height?

(c) Find the maximum height reached by the ball.

(d) What is the velocity of the ball when it is about to hit the ground? What about its acceleration? (Recall that the velocity is precisely the derivative of the position function and the acceleration its second derivative.)

(8) Find all points of the form (x_0, y_0) with $0 \leq x_0 \leq 2\pi$ in the graph of $f(x) = \cos^2(x)$ where the tangent line is horizontal.